Autograms

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Preface:
An *Autogram* is a self-referential statement, that lists the frequencies of its own letters and symbols [1, 2, 3]. I should let one describe itself:

This Autogram contains Fifty-Three spaces, Twenty-Four commas, Five hyphens, Seven As, Four Cs, Three Ds, Thirty-Seven Es, Fifteen Fs, Four Gs, Twelve Hs, Sixteen Is, Two Ls, Four Ms, Seventeen Ns, Thirteen Os, Four Ps, Fifteen Rs, Thirty-Four Ss, Twenty-Six Ts, Eight Us, Eight Vs, Five Ws, Three Xs, Seven Ys, and One period.

I encourage the reader to count the characters above and verify that the sentence is true and the list is comprehensive. However, the real fun with Autograms comes from constructing them. Their premise naturally leads to fun puzzles and brain teasers.

This sentence contains ________ Es.

This sentence contains ________ Os.

In the first example above, either of the words *Four* or *Five* could be inserted in the blank to create a consistent statement. The second example can't be solved just with a number word—placing *Two* in the blank leads to a sentence with Three Os—placing *Three* in the blank leads to a sentence with Two Os. Instead, the reader might choose a more creative solution.

This sentence contains Four Es, and Four Os.

An Autogram is created when *all* of the characters' frequencies are enumerated correctly, or when the list is comprehensive in some way—capitalization or punctuation may be disregarded, and any numbering system may be employed. With these loose guidelines, many languages, list formats, and self-reference structures are possible.

This Autogram contains XLV spaces, XX commas, VIII As, IV Cs, III Ds, III Es, II Gs, II Hs, XXXIV Is, II Ls, IV Ms, IV Ns, V Os, III Ps, III Rs, XXIV Ss, IV Ts, II Us, X Vs, XI Xs, and a period.

In remainder of this article will present some Autograms which I constructed, using various rule sets and premises. My goal in this study is to further popularize the Autogram concept [2], which I find delightfully artistic, thought provoking, and under-explored in puzzle design.

All of the Autograms contained in this article, including those above, were constructed by the present author (aided with a computer and search algorithm). At the end of the article, I provide a few Autogram puzzles which I believe are of reasonable difficulty for a human to complete un-aided.

Anagrams:
I'll start with some shorts, and build in complexity to larger self-referential passages.

2 3s
3 2s
2 5s
5 Ss

Attempting to use few characters, I wrote this statement without punctuation and with arithmetic numerals in place of number words. Nonetheless, there is a nice visual—the second numeral on any line is the same as the first numeral on the next—and I've attempted to highlight this pattern using the staircase layout. Here's another with even fewer characters:

3 3s
3 Ss

And one last one:

twice twice

None of the above employ a sentence prefix like *This sentence contains* or a conjunction like *and* in the list. When an Autogram contains no extraneous words like this, and is comprised only of the character frequency list, it may be called a *Reflexicon* [1].

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Three Xs, Seven Ys, Fifty spaces, Twenty-Five commas, Four hyphens, Three As,
One B, Three Cs, Thirty-Five Es, Six Fs, Two Gs, Ten Hs,
Eleven Is, Two Ls, Three Ms, Nineteen Ns, Nine Os, Three Ps, One Q, Eight Rs,
Twenty-Nine Ss, Twenty-One Ts, Two Us, Six Vs, Seven Ws,
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In an attempt to avoid an incomplete sentence, I wrote this Reflexicon so that it neither starts nor stops. The statement contains *One B* and *One Q*, but no *Ds*, *Js*, *Ks*, or *Zs*. This creates degrees of freedom in the solution—one could replace the *One Q* with *One D* and the statement would remain consistent. Leaving degrees of freedom in this way can produce an unsatisfying Autogram unless there is some other interesting reason why certain symbols appear only once.

When every symbol of an Autogram is used two or more times, that Autogram may be called *Pure* [1]. The first Autogram of this article's preface is a pure; however, it's prefix *This Autogram contains* admits other, impure, solutions.

This Autogram contains Sixty-Three spaces, Twenty-Nine commas, Six hyphens, Seven As, One B, Four Cs, Three Ds, Thirty-Eight Es, Six Fs, Four Gs, Ten Hs, Sixteen Is, One J, One K, Two Ls, Four Ms, Twenty-Six Ns, Seventeen Os, Four Ps, One Q, Twelve Rs, Thirty-Nine Ss, Twenty-Four Ts, Seven Us, Seven Vs, Six Ws, Seven Xs, Eight Ys, One Z, and One period.

Certain characters do appear only once in the Autogram above; however, I have removed degrees of freedom in the solution by requiring that all twenty-six English letters be included. That is, the statement is both an Autogram and a Pangram, and now there are no degrees of freedom. See also [1, 2] on *Pangramic-Autograms.*

Whenever possible, I try to avoid the Third, Seventh, Tenth, Eleventh, Thirteenth, Seventeenth, Twenty-first, and Twenty-sixth letters of the alphabet.

One might call this a *Lipogramic-Autogram*—since the character set is both intentionally limited and described comprehensively. Constructing Lipogramic-Autograms brings its own challenges; one couldn't write *avoids the Fourteenth letter of the alphabet*, for instance, since *N*, the Fourteenth letter of the alphabet, appears in the word *Fourteenth*.

In that sentence, there are exactly Fifty-Six spaces, Twenty-Five commas, Five hyphens, Seven As, Five Cs, Three Ds, Forty-Seven Es, Fifteen Fs, Three Gs, Twelve Hs, Fifteen Is, Three Ls, Three Ms, Eighteen Ns, Ten Os, Four Ps, Fourteen Rs, Thirty-Two Ss, Twenty-Six Ts, Five Us, Ten Vs, Four Ws, Four Xs, Eight Ys, and One period.

In that sentence, there are exactly Fifty-Six spaces, Twenty-Five commas, Five hyphens, Seven As, Five Cs, Three Ds, Forty-Four Es, Sixteen Fs, Three Gs, Twelve Hs, Fifteen Is, Three Ls, Three Ms, Seventeen Ns, Ten Os, Four Ps, Fourteen Rs, Thirty-One Ss, Twenty-Seven Ts, Five Us, Eight Vs, Five Ws, Four Xs, Eight Ys, and One period.

The style above maybe be called an *Autogram Chain* [1]—a set of passages, each one describing its neighbor's character frequency. Just as a sentence prefix like *This Autograms contains* may admit multiple Autogram suffixes, a chain prefix like *In that sentence, there are exactly* may admit multiple chains. Here is a chain of length 3:

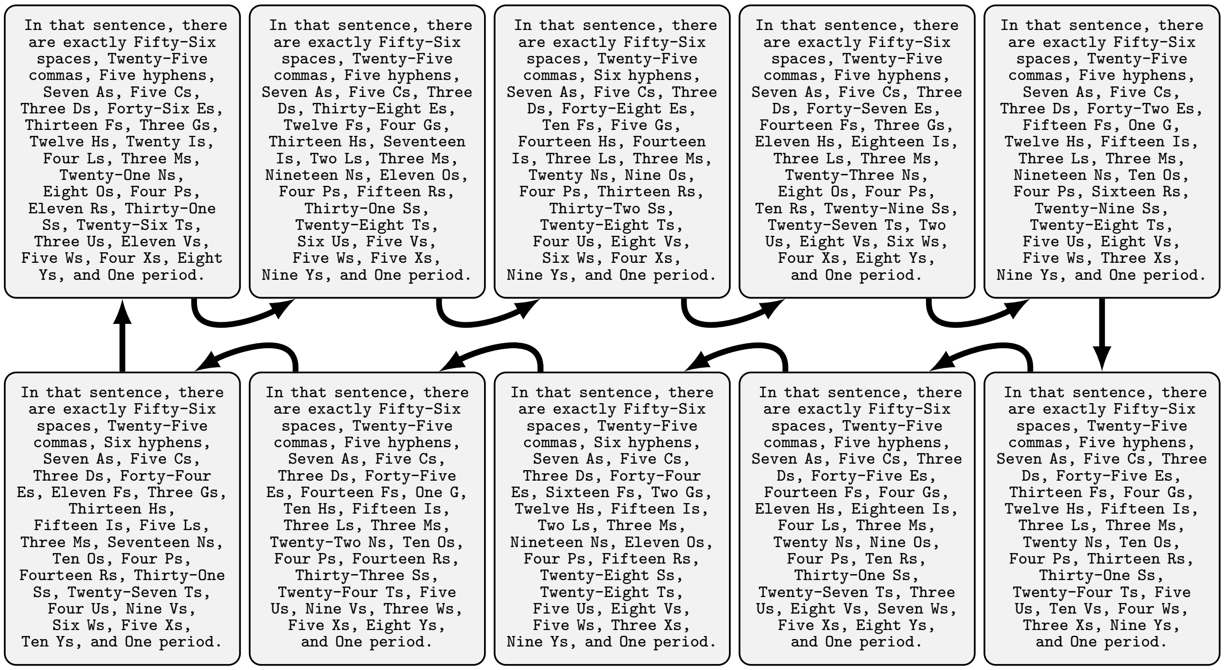
In that sentence, there are exactly Sixty spaces, Twenty-Seven commas, Four hyphens, Seven As, Five Cs, Three Ds, Forty-Five Es, Twelve Fs, One G, Eleven Hs, Sixteen Is, Three Ls, Three Ms, Twenty-Four Ns, Twelve Os, Four Ps, One Q, Fifteen Rs, Thirty Ss, Twenty-Five Ts, Five Us, Ten Vs, Five Ws, Three Xs, Nine Ys, One Z, and One period.

In that sentence, there are exactly Sixty spaces, Twenty-Seven commas, Four hyphens, Seven As, Five Cs, Three Ds, Forty-Five Es, Nine Fs, Two Gs, Eleven Hs, Thirteen Is, Five Ls, Three Ms, Twenty-Three Ns, Thirteen Os, Four Ps, One Q, Fourteen Rs, Thirty Ss, Twenty-Five Ts, Five Us, Nine Vs, Five Ws, Four Xs, Nine Ys, One Z, and One period.

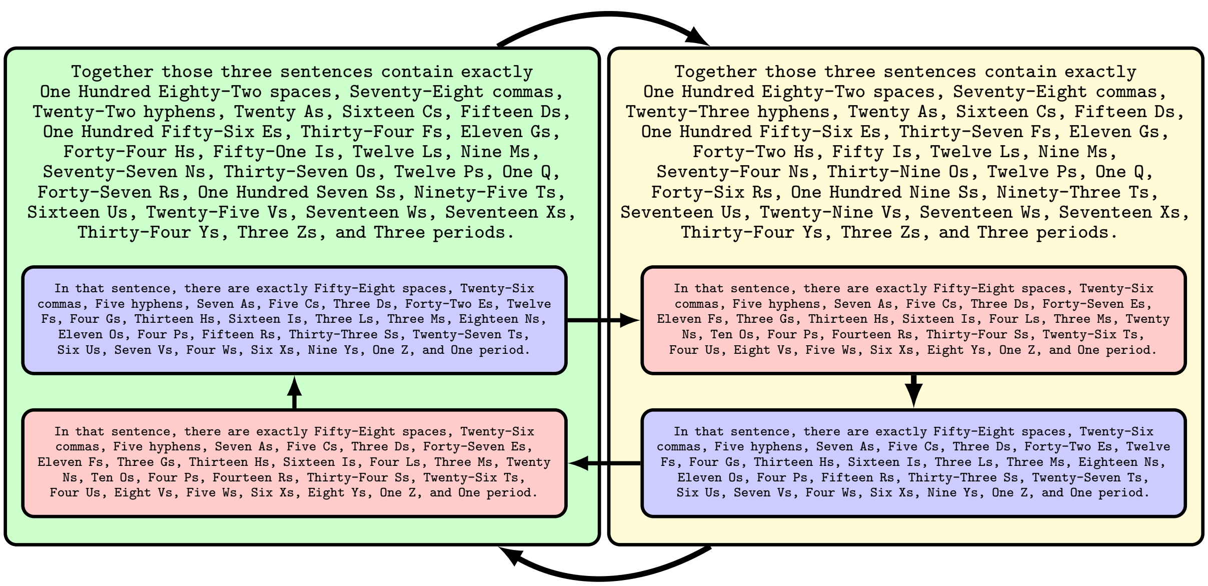
In that sentence, there are exactly Sixty spaces, Twenty-Seven commas, Four hyphens, Seven As, Five Cs, Three Ds, Forty-Eight Es, Twelve Fs, One G, Ten Hs, Thirteen Is, Five Ls, Three Ms, Twenty-One Ns, Eleven Os, Four Ps, One Q, Thirteen Rs, Thirty Ss, Twenty-Five Ts, Four Us, Eleven Vs, Six Ws, Four Xs, Nine Ys, One Z, and One period.

Constructing an Autogram chain is perhaps easier than constructing an stand-alone Autogram—one may start by writing any passage, and then writing the enumerating sentence that describes it—working backwards until a chain emerges. With chains, it's possible to have characters that appear once but do not add degrees of freedom: notice, for example, that the statement *One G* appears in the upper passage and lower left passage, whereas *Two Gs* appears in the lower right passage. While the character *G* does not create degrees of freedom in the solution, the fact that *One Z* appears in each sentence does.

Given any chain, one can construct a larger chain by reusing sentences multiple times—for example, using our chain of length 3, we might create chains of length, e.g., 6, 9, 12, ... by reusing the same sentence set over and over again. I'll call a chain *Reduced* if each sentence within the chain is unique. Any single sentence in a reduced chain defines the whole chain—that is, the same sentence will not appear in more than one reduced chain. For this prefix *In that sentence, there are exactly*, I was able to find reduced chains of length 2, 3, 5, 6, 7, 8, 9, 10 and 1.



Below, I provide a final chain example where I construct chain of length 4, by repeating a chain of length 2 twice. Then I created a chain of length 2 surrounding the chain of legnth 4. I call this chain *Composite* since there is a chain contained within a chain, and the outer chain here is reduced.



Closing Remark:
As I hope the above has made clear, Autograms provide a large and under-explored landscape for which to be creative. I hope this article serves as encouragement for you to attempt to construct your own.

Puzzles:
For each of the following, fill in the blanks so that every character that appears in the completed sentence is enumerated correctly within the sentence. Every blank must contain only numerals, and no character appears more than 100 times. A video-solution for the first puzzle is available at **https://youtu.be/cSFNeUEL7qI**.

This sentence contains __49__ spaces, __22__ commas, __6__ As, __5__ Cs, __3__ Ds, __6__ Es, __2__ Hs, __4__ Is, __3__ Ms, __6__ Ns, __4__ Os, __3__ Ps, __2__ Rs, __27__ Ss, __4__ Ts, ______ 2s, ______ 3s, ______ 4s, ______ 5s, __4__ 6s, ______ 7s, ______ 9s, and a period.

This sentence contains __49__ spaces, __22__ commas, __6__ As, __5__ Cs, __3__ Ds, __6__ Es, __2__ Hs, __4__ Is, __3__ Ms, __6__ Ns, __4__ Os, __3__ Ps, __2__ Rs, __27__ Ss, __4__ Ts, ______ 2s, ______ 3s, ______ 4s, ______ 5s, __5__ 6s, ______ 7s, ______ 9s, and a period.

This sentence contains __49__ spaces, __22__ commas, __6__ As, __5__ Cs, __3__ Ds, __6__ Es, __2__ Hs, __4__ Is, __3__ Ms, __6__ Ns, __4__ Os, __3__ Ps, __2__ Rs, __27__ Ss, __4__ Ts, ______ 2s, ______ 3s, ______ 4s, ______ 5s, __6__ 6s, ______ 7s, ______ 9s, and a period.

References

[1] "Autograms: Self-enumerating sentences," 2023, accessed: 2023-12-09. [Online]. Available: https://autograms.net/

[2] L. Sallows, "In quest of a pangram," *Abacus*, vol. 2, no. 3, pp. pp. 22 – 40, 1985. [Online]. Available: https://leesallows.com/files/In%20Quest%20of%20a%20Pangram1.pdf

[3] E. Wassenaar, "Self-enumerating pangrams: A logological history," 1999, accessed: 2023-12-09. [Online]. Available: https://www.fatrazie.com/jeux-de-mots/recreamots/287-self-enumerating-pangrams