Deal Breaker
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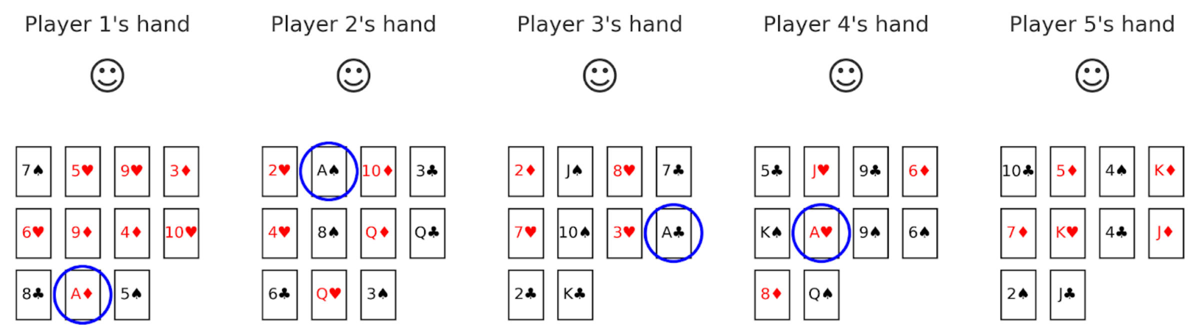
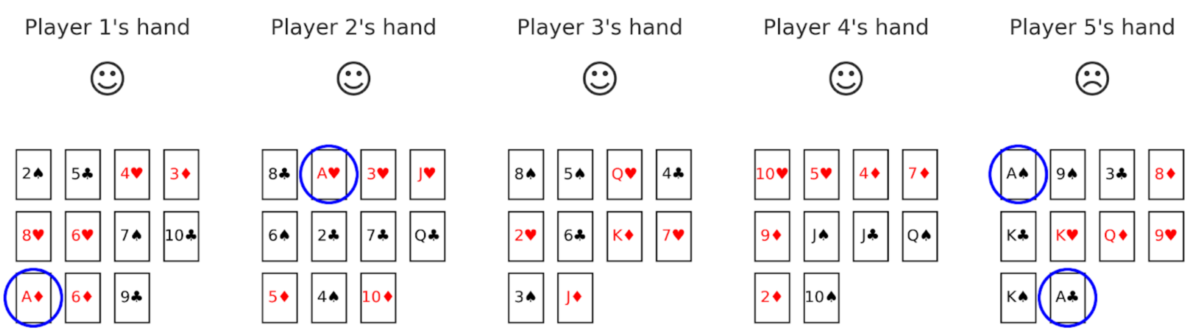
Imagine dealing a shuffled deck of 52 cards to five players. All of the cards are distributed, so that two players receive eleven cards, and the rest receive ten.

Question: What is the probability that, after the deal, no player ends up with more than one Ace?



No player has more than one Ace

WIN



Player 5 has more than one Ace

LOSE

Probability of win: ____ / ____

Express your answer as a **reduced fraction**

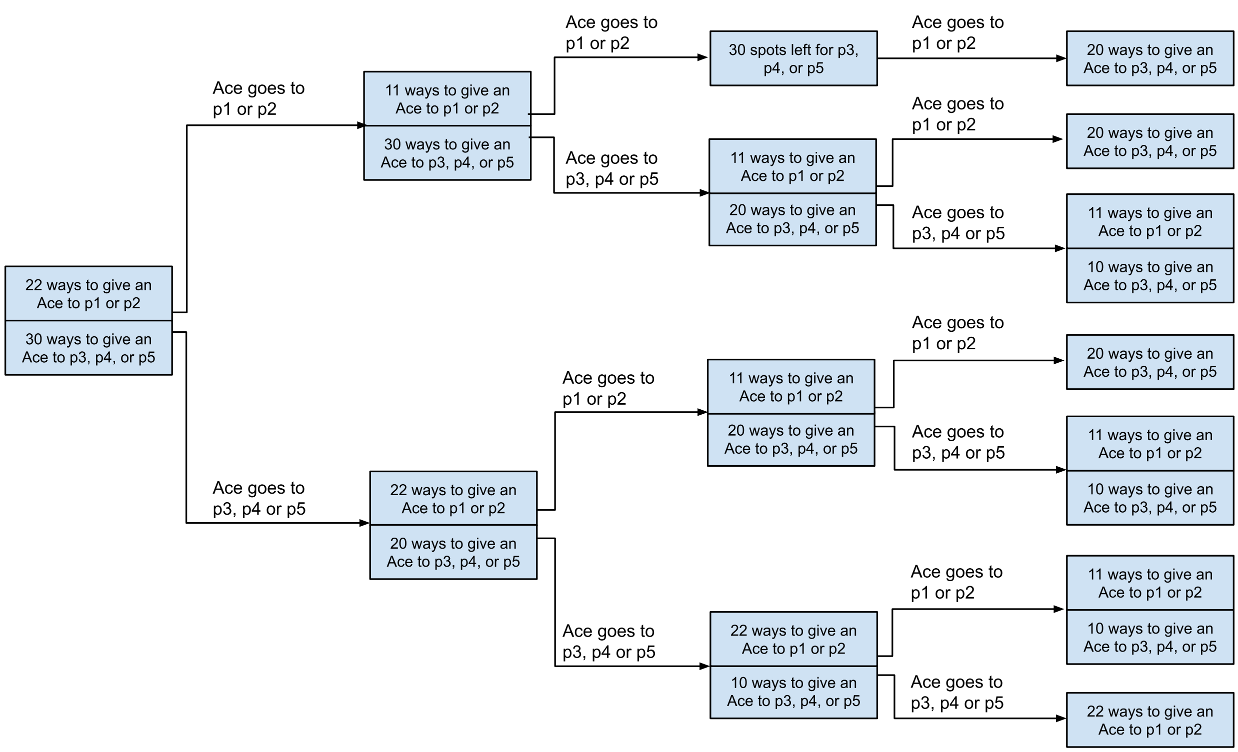
Note: The following is how I solved the problem initially, although I was impressed later to find a more elegant solution, produced by chatGPT (o4-mini-high with deep research). From what it seems, chatGPT was able to reason and solve this puzzle on its own, without finding a solution online or using any other outside help: [View the chat](#).

Solution

Think of the deck as having 52 spots for cards. First compute the total number of ways the four Aces to be placed in the 52 spots:

$$\text{Total Aces placements} = 52 \cdot 51 \cdot 50 \cdot 49.$$

Then compute the total number of ways the four Aces to be placed in the 52 spots, so that no one player receives more than one Ace. Notice that the first Ace can be placed in any of the 52 spots, and the only consideration that needs to be carried further is whether the spot belongs to players 1 and 2 (who each receive 11 cards), or players 3, 4 and 5 (who each receive 10). If, for example, the first Ace is given to players 1 and 2 (there are 22 spots that create this scenario), then there will be 41 remaining slots for the second Ace. If the second Ace then goes to players 3, 4 and 5 (there are 30 spots that create this scenario), then there will be 31 remaining slots for the second Ace. Using very light brute force, it is possible to play through all scenarios.



Example:

- First Ace goes to player 1 or 2 (22 spot options in deck)
- Second Ace goes to player 3, 4, or 5 (30 options)
- Third Ace goes to player 3, 4, or 5 (20 options)
- Fourth Ace goes to player 1 or 2 (11 options)

Total number of ways this can happen: $22 \cdot 30 \cdot 20 \cdot 11$.

To arrive at the total number of ways the four Aces to be placed in the 52 spots, so that no one player receives more than one Ace, it is possible to compute the number of deals which create each path in the figure, and then sum across all possible paths.

$$
\begin{array}{lr}
 & 22 \cdot 11 \cdot 30 \cdot 20 \\
 & 22 \cdot 30 \cdot 11 \cdot 20 \\
 & 22 \cdot 30 \cdot 20 \cdot 11 \\
 & 22 \cdot 30 \cdot 20 \cdot 10 \\
 & 30 \cdot 22 \cdot 11 \cdot 20 \\
 & 30 \cdot 22 \cdot 20 \cdot 11 \\
 & 30 \cdot 22 \cdot 20 \cdot 10 \\
 & 30 \cdot 20 \cdot 22 \cdot 11 \\
 & 30 \cdot 20 \cdot 22 \cdot 10 \\
+ & 30 \cdot 20 \cdot 10 \cdot 22 \\
\hline
 & 1399200
\end{array}
$$

Now the probability that no one player receives more than one ace in the Deal is given by

$$\frac{1399200}{52 \cdot 51 \cdot 50 \cdot 49} = \frac{2332}{10829} \approx 21.535\%$$

This is the solution to the puzzle.