Math Rules
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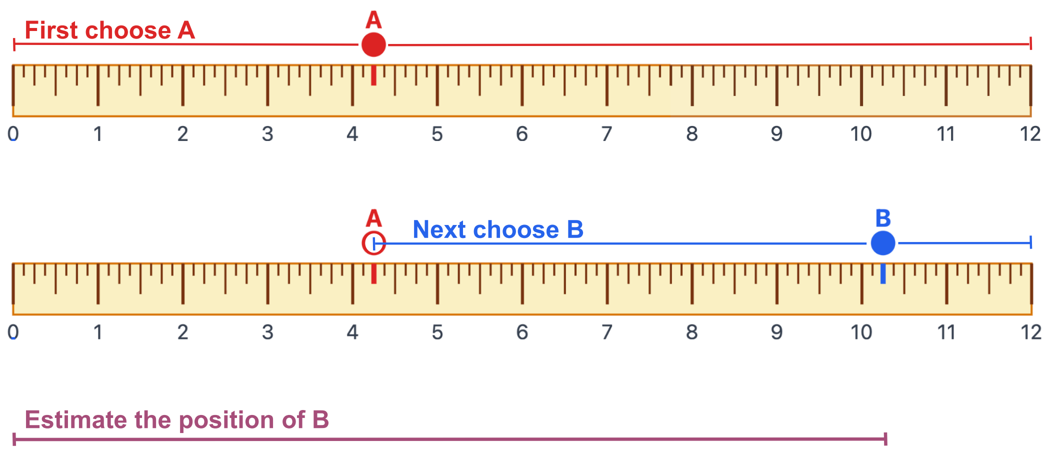
Part 1.

A 12-inch ruler has tick marks every 1/8 inch.

First, <u>choose one tick mark at random</u> from the ruler, anywhere between 0 inches and 12 inches (each tick mark is equally likely). Call this point A.

Next, <u>choose one tick mark at random</u> from the tick marks between A and 12 inches (including both A and 12). Call this point B.

On average, how far is B from the 0-inch mark? _____



Part 2.

Now suppose the ruler has no tick marks at all.

First, choose <u>a point at random</u> between 0 inches and 12 inches (every position is equally likely). Call this point A.

Next, choose <u>a point at random</u> between A and 12 inches (including both endpoints). Call this point B.

On average, how far is B from the 0-inch mark? _____

Solution

Part 1 and 2: In both cases, B will land <u>9 inches</u> from the 0-inch mark.

Part 1.

In the first step, there are 97 possible places for A to land, $\frac{0}{8}, \frac{1}{8}, \frac{2}{8}, \ldots \frac{96}{8}$ and each has a probability of $P[\,A] = \frac{1}{97}$. Given the fact that A lands on spot $\frac{n}{8}$ (for some $n \in \{0, 1, \ldots, 96\}$) the expected value of the final location of B will be the midpoint between A and the end of the ruler, $E[\, location\, B \mid A = \frac{n}{8}\,] = \frac{1}{2}(\frac{n}{8} + 12)$. Therefore, the expected location of B will be

$$E[\, location\, B\,] = \sum_{n=o}^{96} E[\, location\, B \mid A = \frac{n}{8}\,]\, P[A = \frac{n}{8}] = \sum_{n=o}^{96} \frac{1}{97}\frac{1}{2}(\frac{n}{8} + 12) \;= 9\, in.$$

This is the solution to Part 1.

Part 2.

In the first step, the location of A is chosen uniformly between the rulers endpoints, meaning the probability density function is $P[A = x] = \frac{1}{12}$ where $x \in [0\, in., 12\, in.]$. Given the fact that A lands on spot x, the expected value of the final location of B will be the midpoint between x and B, $E[\, location\, B \mid A = x\,] = \frac{1}{2}(x + 12)$ in. Therefore, the expected location of B will be

$$E[\, location\, B\,] = \int_0^{12} E[\, location\, B \mid A = x\,] P[A = x]\, dx = \int_0^{12} \frac{1}{12}\frac{12-x}{2}\, dx = 9\, in.$$

This is the solution to Part 2.