

Forty-Five

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How many distinct sets of consecutive integers sum to 45?

Example: $(a, b) = (14, 16)$ solves

$$\sum_{k=a}^b k = 45 \quad (1)$$

since $14 + 15 + 16 = 45$.

How many distinct pairs (a, b) exist? _____

Solution:

There are 12 ways to construct 45 as the sum of consecutive integers:

1.	$(a, b) = (45, 45)$	$45 = 45$	(1 term)
2.	$(a, b) = (22, 23)$	$45 = 22 + 23$	(2 terms)
3.	$(a, b) = (14, 16)$	$45 = 14 + 15 + 16$	(3 terms)
4.	$(a, b) = (7, 11)$	$45 = 7 + 8 + 9 + 10 + 11$	(5 terms)
5.	$(a, b) = (5, 10)$	$45 = 5 + 6 + 7 + 8 + 9 + 10$	(6 terms)
6.	$(a, b) = (1, 9)$	$45 = 1 + 2 + \dots + 8 + 9$	(9 terms)
7.	$(a, b) = (0, 9)$	$45 = 0 + 1 + \dots + 8 + 9$	(10 terms)
8.	$(a, b) = (-4, 10)$	$45 = -4 + -3 + \dots + 9 + 10$	(15 terms)
9.	$(a, b) = (-6, 11)$	$45 = -6 + -5 + \dots + 10 + 11$	(18 terms)
10.	$(a, b) = (-13, 16)$	$45 = -13 + -12 + \dots + 15 + 16$	(30 terms)
11.	$(a, b) = (-21, 23)$	$45 = -21 + -20 + \dots + 22 + 23$	(45 terms)
12.	$(a, b) = (-44, 45)$	$45 = -44 + -43 + \dots + 44 + 45$	(90 terms)

This is the solution to the puzzle.